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Quasi-Parabolic Technique for Spatial Marching Navier-Stokes Computations

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Abstract

A COMPUTATIONAL technique is presented for obtaining flowfield solutions to a parabolic form of the Navier-Stokes equations. The point of departure is the general interpolants method (GIM), which provides a discretization for partial differential equations on arbitrary three-dimensional geometries. The new scheme, termed quasi-parabolic (QP), treats the parabolized equations but with "time-like" terms appended. Addition of these extra terms, which are relaxed by iteration, avoids many of the singularities inherent in classical parabolic Navier-Stokes methods. Solutions are presented for viscous flows in boundary layers and free shear layers as computed with the GIM/QP scheme.

Contents

The full three-dimensional Navier-Stokes equations are elliptic in character and require either time-dependent or relaxation/iteration schemes to integrate the complete spatial flowfield simultaneously. This can require large amounts of computer storage and relatively long run times. For situations in which a region of the flow is inviscid and entirely supersonic, a spatial hyperbolic marching algorithm would be efficient. There are also many viscous problems of interest in which parabolic marching solutions are acceptable.

Certain assumptions must be made in using a spatial marching technique. There must exist a dominant flow direction in which to march, and there can be no flow back upstream, i.e., no recirculation in the streamwise direction. Stress terms are not allowed to act on the cross planes, i.e., there can be no second-order terms (diffusion, viscosity) in the marching coordinate. The downstream pressure field also must not be allowed to propagate upstream. Reference 1 contains a review of the approaches to the solution of the parabolic Navier-Stokes (PNS) equations. The paper of Lin and Rubin[2] also discusses many of the approaches and difficulties involved in PNS solutions.

The intent of this research is to provide a parabolic spatial marching technique which can be readily incorporated into the general interpolants method (GIM). The GIM code provides a discretization of partial differential equations on arbitrary three-dimensional geometries. The GIM treatment of arbitrary geometric shapes in three dimensions and the method of solution of the full Navier-Stokes equations is described fully in Refs. 1, 3, and 4 and will not be repeated here.

Three basic requirements were placed on a GIM/marching algorithm: 1) the geometric treatment must be applicable to arbitrary shapes; 2) the same basic algorithm should be applied to both hyperbolic and parabolic flows and be capable of eventual coupling with an automated algorithm for switching between parabolic and elliptic solvers; and 3) the algorithm should be readily vectorizable to realize the speed gain from using the CYBER 203 computer.

The basic idea of the GIM technique is to combine the classical parabolic marching approach with a "quasi-time" relaxation. The parabolic-march procedure greatly reduces the amount of computer storage compared to a fully elliptic field. The time relaxation form of the equations eliminates the decode ambiguity associated with mixed subsonic/supersonic flows, allows velocity boundary conditions at solid walls to be treated, and allows inclusion of the streamwise pressure gradient term. The QP backward difference scheme does not avoid departure solutions but it has proved to be less sensitive to streamwise pressure gradients than classical PNS approaches. Lin and Rubin[2] also introduce pseudo-time relaxation terms in a space marching method using a linearized implicit finite difference procedure. The GIM technique, although developed independently of Lin and Rubin, also employs time relaxation but with an explicit finite difference scheme and arbitrary three-dimensional geometries. The second-order backward-forward/backward-backward explicit predictor-corrector scheme of the GIM code is also a unique approach to parabolic marching solutions.

The equations used in the QP method are the time-averaged full Navier-Stokes written in three-dimensional Cartesian coordinates in conservation form, but with all second-order terms dropped from the flux vector in the quasi-marching coordinate. Included are global mass conservation, three components of momentum, and total energy conservation. These will be termed the quasi-parabolic Navier-Stokes equations (QPNS). Another way to view the QPNS equations is to take the parabolized Navier-Stokes and reintroduce pseudo-time derivatives. Thus the QP algorithm is not a classical space marching scheme, and is also not a time-dependent elliptic method. It is somewhat of a hybrid technique which combines the better features of two approaches and eliminates some of the less desirable ones.

Boundary conditions for the QPNS equations are those typical of the elliptic system: known "inflow" boundaries, no-slip viscous walls, free-slip tangency for inviscid walls, constant temperature and adiabatic thermal boundary conditions, and other nonstandard boundaries which are all described in detail in Refs. 1, 3, and 4. The system is formally closed with a choice of constant, Sutherland's law laminar-type, or algebraic turbulent eddy viscosity-type values for gas properties.

The QP solution procedure, as any parabolic marcher, allows no streamwise diffusion effects. The solution is assumed known at upstream data planes, $1, 2, \ldots, K-1$, and the solution is sought at plane K with no knowledge of plane $K+1$. Quasi-time relaxation is used to obtain the solution at plane K in terms of the (converged) solution at a number of upstream data planes. Second-order backward streamwise differences are used to prohibit downstream feedback and the cross plane operators use an alternating forward-backward predictor-corrector sweep. A two-step sequence is used to implement the difference scheme in the spatial dimensions.

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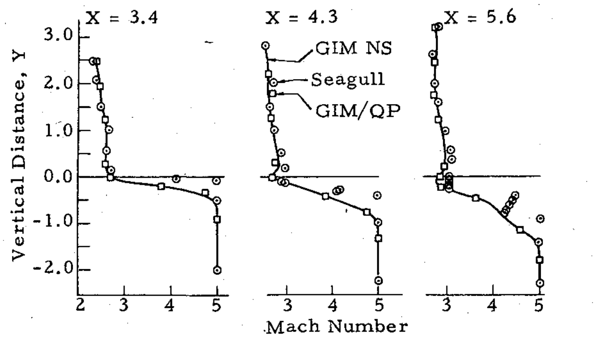

Fig. 1 Shear flow computation—comparison of GIM/QP with a full Navier-Stokes code[3] and a forward-marching slip line code.[5]

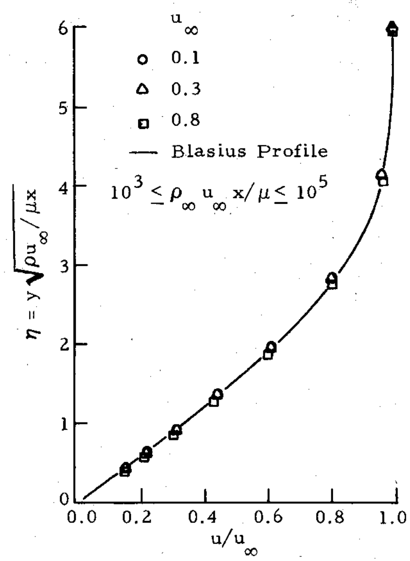

Fig. 2 Comparison of GIM/QP boundary layer with classical Blasius profile.

Stability of this explicit scheme is obtained by obeying the classical CFL constraint of explicit methods. Truncation error control is maintained by using the numerical diffusion cancellation scheme.[3]

The procedure for marching a solution is outlined as follows: The upstream planes are known values, i.e., the converged previous QP step, and thus serve the role of "inflow boundaries." The plane of interest K plays the role of an "outflow boundary" whose values are determined from quasi-time relaxation to steady-state via iteration of the predictor-corrector scheme. The initial conditions for this procedure are provided by starting each downstream plane equal to the converged solution at the plane just upstream. This provides a reasonable guess if the planes are not spaced too far apart. Upon convergence, the conservation variables are decoded and the plane K is set $K+1$. The marching procedure is then repeated. Details of the marching procedure, the finite difference scheme, and the convergence criteria used in the GIM/QP technique are given in Ref. 1.

The algorithm just described is coded in vector FORTRAN for the CDC CYBER 203 machine at NASA Langley Research Center. A number of example calculations have been made to check out the coding, test convergence, and determine the ability of the algorithm to handle inviscid and viscous flows.[1] Two cases are presented for illustration of the technique. The first case consists of two-dimensional viscous flow resulting from interaction of a nozzle exhaust with a hypersonic freestream. The QP algorithm gives results virtually identical to those given by the full Navier-Stokes code.[3] Comparison of these solutions and with the inviscid SEAGULL code[5] are shown in Fig. 1 as vertical Mach number distributions at three axial stations in the shear region. As seen by the comparisons, the GIM marching algorithm does indeed work as expected, gives quantitatively the same answers as the other codes, and can accurately calculate parabolic flows of the free shear layer type.

Case 2 is a calculation of the external laminar flow over a flat plate of unit length at freestream Mach numbers $M_\infty = 0.1$, 0.3, and 0.8, Prandtl number $Pr = 0.7$, Reynolds number per unit length $Re_\infty = 10^5$, and freestream pressure $P_\infty = 1.0$. This case was run to verify the ability of the technique to compute the subsonic flow in the vicinity of a no-slip, adiabatic wall. Figure 2 shows the results of the calculation in terms of the Blasius similarity parameters. The agreement with the Blasius analytic curve is excellent. The solutions at all planes varied only by about 0.5% after such scaling and could not be distinguished on the figure.

The computer code is currently being used to compute viscous flow in an aircraft inlet and three-dimensional viscous flow over a missile at incidence. Inclusion of a linearized-block-implicit difference algorithm is also underway.

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